Wavelets and Time-Frequency Decomposition

(Syllabus)

(Institute Elective for UG/PG 3-1-0)

Fundamentals of Linear Algebra:

Vector spaces, Bases, Orthogonality, Orthonormality, Projection, Functions and function spaces

Orthogonal functions, Orthonormal functions, Orthogonal basis functions

Signal Representation in Fourier Domain

Fourier series, Orthogonality, Orthonormality and the method of finding the Fourier coefficients Complex Fourier series, Orthogonality of complex exponential bases, Mathematical preliminaries for continuous and discrete Fourier transform, limitations of Fourier domain signal processing.

Short Time Fourier Transform (STFT):

Signal representation with continuous and discrete STFT, concept of time-frequency resolution, Resolution problem associated with STFT, Heisenberg's Uncertainty principle and time frequency tiling, Why wavelet transform?

Introduction to Wavelet Transform:

The origins of wavelets, Wavelets and other wavelet like transforms, History of wavelet from Morlet to Daubechies via Mallat, Different communities and family of wavelets, Different families of wavelets within wavelet communities

Continuous Wavelet Transform:

Wavelet transform-A first level introduction, Continuous time-frequency representation of signals, Properties of wavelets used in continuous wavelet transform, Continuous versus discrete wavelet transform

Discrete Wavelet Transform:

Haar scaling functions and function spaces, Translation and scaling of $\phi(t)$, Orthogonality of translates of $\phi(t)$, Function space V_0 , Finer Haar scaling functions, Concepts of nested vector spaces, Haar wavelet function, Scaled and translated Haar wavelet functions, Orthogonality of $\phi(t)$ and $\psi(t)$, Normalization of Haar bases at different scales, Refinement relation with respect to normalized bases, Support of a wavelet system, Daubechies wavelets, Plotting the Daubechies wavelets,

Designing Orthogonal Wavelet Systems-A Direct Approach:

Refinement relation for orthogonal wavelet systems, Restrictions on filter coefficients, **Condition-1:** Unit area under scaling function, **Condition-2:** Orthonormality of translates of scaling functions, **Condition-3:** Orthonormality of scaling and wavelet functions, **Condition-4:** Approximation conditions (Smoothness conditions), Designing Daubechies orthogonal wavelet system coefficients, Constraints for Daubechies' 6 tap scaling function.

Discrete Wavelet Transform and Relation to Filter Banks:

Signal decomposition (Analysis), Relation with filter banks, Frequency response, Signal reconstruction: Synthesis from coarse scale to fine scale, Upsampling and filtering, Perfect reconstruction filters, QMF conditions, Computing initial s_{j+1} coefficients, Concepts of Multi-Resolution Analysis (MRA) and Multi-rate signal processing.

Biorthogonal Wavelets:

Biorthogonality in vector space, Introduction to Biorthoeonal Wavelet Systems, Signal Representation Using Biorthogonal Wavelet System,

Wavelet Packets and M-Band Wavelets:

Wavelet Packet Analysis: Signal representation using Wavelet Packet Analysis, Selection of best basis, Introduction of M-Band wavelet system, Signal representation using M-Band wavelet systems.

Applications of Wavelets:

Applications of wavelets in signal and image processing and other related engineering Fields.

Text/Reference Books:

- K. P. Soman, K. I. Rmachandran, N. G. Resmi, "Insight into Wavelets: From Theory to Practice, (Third Edition)", PHI Learning Pvt. Ltd., 2010.
- A.N. Akansu and R.A. Haddad, "Multiresolution signal Decomposition: Transforms, Subbands and Wavelets", Academic Press, Oranld, Florida, 1992.
- John G. Proakis, Dimitris G. Manolakis, "Digital Signal Processing", Pearson Prentice Hall, 2007.

- Rafael C. Gonzalez, Richard E. Woods "Digital Image Processing (Third Edition)", Pearson International Edition, 2009.
- C. S. Burrus, Ramose and A. Gopinath, Introduction to Wavelets and Wavelet Transform, Prentice Hall Inc.

Wavelet links:

1. http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html
2. http://www.wavelet.org/
3. http://www.math.hawaii.edu/~dave/Web/Amara's%20Wavelet%20Page.htm

Objectives:

Wavelets have established themselves as an important tool in modern signal processing as well as in applied mathematics. The objective of this course is to establish the theory necessary to understand and use wavelets and related constructions. A particular emphasis will be put on constructions that are amenable to efficient algorithms, since ultimately these are the ones that are likely to have an impact. We thus study applications in signal and image processing where time-frequency transforms like wavelets play an important role.

Course Objectives: Upon completion of this course, students should be able to:

1. Understand the terminology that are used in the wavelets literature.
2. Explain the concepts, theory, and algorithms behind wavelets from an interdisciplinary perspective that unifies harmonic analysis (mathematics), filter banks (signal processing), and multiresolution analysis (computer vision).
3. Understand how to use the modern signal processing tools using signal spaces, bases, operators and series expansions.
4. Apply wavelets, filter banks, and multiresolution techniques to a problem at hand, and justify why wavelets provide the right tool.
5. Think critically, ask questions, and apply problem-solving techniques.